



Ganglion startup in porous media

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ABSTRACT

In porous media, dispersed fluid ganglia (bubbles, droplets) largely shape flow, transport, and chemical reactions. Static trapping and steady-state mobilization of ganglia have been extensively studied. However, the transition from trapping to mobilization (ganglia startup) has received less attention. Here we investigate ganglia startup in a 2-D uniform porous medium subject to an external field. When the external field quasi-statically increases from zero, a ganglion first irreversibly and discretely reformulates without mobilization; when the external field reaches a criterion, the ganglion starts to mobilize by sequential non-equilibrium jumps. Surprisingly, startup significantly reduces ganglia's capillary hysteresis: although a static ganglion with a fixed volume has multiple metastable morphologies, it normalizes into one "kick-off" state before startup, at which state the ganglion's morphological and thermodynamical properties fall into very narrow range regardless of volume. It allows analytical resolution of the startup criterion and may largely simplify multiphase fluids modeling in porous media.

1. Introduction

Ganglia widely emerge in engineering and natural porous media. Nonwetting ganglia can be trapped by capillary forces at the pore-throat, i.e., capillary trapping (Huppert and Neufeld, 2014; Sun, 2016). Capillary trapping of ganglia reshapes flow channels (Tuller and Or, 2001; Haggerty, 2023), thereby significantly affecting flow, mass & heat transport, and chemical reactions (Liu and Wang, 2022; Shahraeeni and Or, 2012; Chen, 2018) in the medium. To mobilize a trapped ganglion in porous media, an external driving force above a threshold is required to overcome the capillary pressure barrier (Larson et al., 1977; Larson et al., 1981; Datta et al., 2014; Yeganeh, et al., 2016). Trapping and mobilization of ganglia is associated with non-linear effects in multiphase flow through porous media which has not been fully understood (Tallakstad, 2009; Tallakstad, 2009; Zhang, 2021; Sinha, 2021; Zhang et al., 2022; Liu, 2022; Anastasiou, 2024), and has become a critical issue in a variety of industrial and natural applications, including in-situ air sparging for the remediation of volatile contaminants (Bjerg, 2002; Pleasant, 2014), the ebullition of gas ganglia from sediments (Baird, 2004; Kellner, 2006), fluidized beds up-flow operation (Tsuchiya et al., 1990; Yang et al., 2007), geological CO₂ sequestration (Huppert and Neufeld, 2014; Juanes, 2006), and enhancement of oil recovery through injecting gas, emulsion, or foam (Føyen, 2020; Géraud, 2016).

On static ganglia trapped by capillary force in porous media, major efforts have been made to characterize their morphologic, hydrodynamic, and thermodynamic properties (McClure, 2016; Wang et al., 2021). It has been theoretically demonstrated that the state of a ganglion in a porous medium is not only determined by its volume, V . Instead, $d + 1$ Minkowski functionals are required to fully characterize a two-phase immiscible system in a d -dimensional system (Hadwiger, 1957; McClure, 2018; Armstrong, 2018). Unfortunately, it is technically very difficult to acquire other Minkowski functionals than V in porous media. If people simply use V as the only variable to characterize the ganglion, the capillary hysteresis effect emerges that one specific V corresponds to multiple possible ganglion states. It seriously challenges the modeling of multiphase fluid behavior in porous media (Haines, 2009; Morrow and Harris, 1965; Hilfer, 2006; Yun and Pantazidou, 1998).

On flowing ganglia in porous media under an external field, the steady-state (terminal) velocity is extensively studied (Roosevelt and Corapcioglu, 1998; Corapcioglu et al., 2004; Ghasemian et al., 2019; Liu, 2019; Zhang et al., 2021). Roosevelt and Corapcioglu (Roosevelt and Corapcioglu, 1998) conduct the first study to measure the terminal velocity of a bubble rising in porous media through video recordings, followed by theoretical work adopting steady-state force balance analysis (Corapcioglu et al., 2004; Cihan and Corapcioglu, 2008). Ghasemian et al. (Ghasemian et al., 2019) conduct micromodel experiments

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and pore network modeling to investigate the bubble migration velocity in a uniform pore network, that looks into the dependency of terminal migration velocity on bubble length and the incline angle. Their models are based mainly on the Darcy-scale equations that overlook pore-scale discontinuity and non-uniformity, although pore volume is always not negligible compared to the ganglia volume. Liu et al. (Liu, 2019; Liu, 2017) conduct experiments and summarize three types of bubble migration mechanisms according to the incline angle, porosity, and pore-throat ratio. Abovementioned studies on ganglion mobilization pay attention to the time-average or steady-state behaviors.

However, very few studies focus on how an initially trapped ganglion (stranding ganglion) in porous media starts up. It is well-recognized that the ganglion becomes mobilized when the external driving force exceeds some capillary pressure barrier (Larson et al., 1977; Datta et al., 2014; Larson et al., 1981; Chatzis and Morrow, 1984). However, the correlation between the driving force and capillary pressure barrier is not trivial, due to the hysteresis effect mentioned above. In a two-dimensional (2-D) uniform porous media, for example, the capillary pressure (P_c) of a ganglion is determined not only by V , but also by Euler characteristics (χ), and pore occupancy (n) (Wang et al., 2021); similarly, the driving force is also correlated with n , χ , and possibly other parameters. As a result, even when V and external field strength are fixed in a porous medium, the capillary barrier and the driving force still cannot be determined. In addition, the topology of the ganglion may undergo significant changes before hydrodynamic startup (Ng et al., 1978; Ng and Payatakes, 1980). Encountering such complicated problems, previous studies adopt some major simplifications. In dealing with P_c , the drainage and imbibition capillary pressure model is adopted for simplicity (Datta et al., 2014; Payatakes et al., 1980), which is valid for the continuous fluid rather than dispersed ganglia (Wang et al., 2021). Additionally, the number of pores occupied by a ganglion (n) was set to be constant if no breakup or coalescence occurs, by assuming that sudden jumps happen simultaneously in the head and the tail (Ng and Payatakes, 1980; Payatakes, 1982), which actually conflicts against experimental and numerical observations (Shams, 2021; Liu and Wang, 2022).

Accordingly, the physics of ganglia startup is still largely unexplored, which is a gap (between ganglia trapping and steady-state mobilization) that needs a bridge to fully understand dispersed fluid behaviors in

porous media. In this work, we conduct a theoretical analysis in a 2-D conceptual porous medium to demonstrate the rich physics behind ganglia startup. We aim to find a theory for the startup criteria and to reveal the pre-startup behaviors.

2. Method

2.1. Model setup

We inherit a conceptual model describing a ganglion residing in a 2-D uniform porous medium. The medium consists of a rectangular array of identical circular grains. We assume an isothermal system, and impose a uniform external field to drive the ganglion. The presence of an external field results in different pressures in the ambient fluid upstream and downstream, which requires varying surface curvature along the ganglia to keep global force equilibrium. Therefore, as shown in Fig. 1a, the morphology of a ganglion is shaped by both the geometric confinement as well as the external field (Wang et al., 2021).

We define the void space enclosed between four adjacent grains a *pore* and the narrow constriction that connects two neighboring pores a *throat*. The pore occupancy, n , refers to the number of pores occupied (fully or partially) by the ganglion. The topology of the ganglion is quantified by the Euler characteristic χ , which is defined as $\chi = \beta_0 - \beta_1 + \beta_2$, where β_i are called Betti numbers (more details are available in (Wang et al., 2021)). We denote the radius of grains by R_1 and the half-distance between the centers of two adjacent grains by R_0 . In subsequent analyses, we set the pore-throat ratio $R_0/(R_0 - R_1) = 6$ for demonstration. The volume and solid surface area of one pore are denoted by V_{pore} and A_{pore} , respectively. Similarly, the volume and surface area of a ganglion are denoted by V and A , respectively.

In the main text, we only consider the “linear” ganglion. “Linear” means that the ganglion does not have any branches and is parallel to the external pressure gradient. It should be noted that a ganglion of complex shapes with branches will eventually evolve into a linear ganglion/ganglia before it starts up, as analyzed in Supporting Information S1. In brief, the ganglia of $\chi < 1$ will eventually evolve to $\chi = 1$ or split up into multiple ganglia that are each of $\chi = 1$. Furthermore, we show that in most cases, a ganglion of $\chi = 1$ will eventually reshape into a linear ganglion without splitting up. Hence, any ganglia will eventually evolve

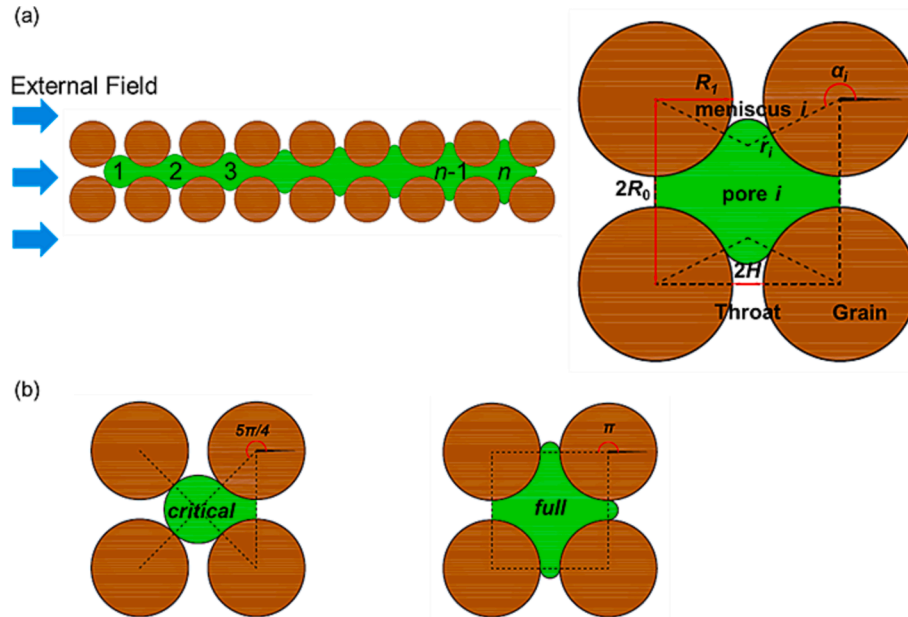


Fig. 1. (a) A 2-D uniform porous medium comprised of an ordered array of identical circular grains. A linear ganglion resides in an external pressure gradient, including a zoom-in to a portion of it. Responding to the external pressure gradient, menisci upstream retract while menisci downstream stretch. (b) The *critical* state and the *full* state of a ganglion. Both states are unstable and result in *flinch* and *breakthrough*, respectively.

into linear ganglia before mobilization. Therefore, this study concentrates on the startup of linear ganglia, while the dynamic process of breakup of ganglia with branches is beyond our scope, with rich approaches reported in the literature (Zhang et al., 2021; Ng and Payatakes, 1980; Constantinides and Payatakes, 1991; Singh, 2017).

The ganglion in our model is assumed to be incompressible and perfectly nonwetting (Melrose and Brandner, 1974). Normal stress and shear stress acting on the interface are calculated that determine the ganglion status and morphology (Ng et al., 1978; Rao and Wong, 2018; Yiotis et al., 2013; Yiotis, 2019). We assume a uniform external pressure gradient (such as a gravitational field). The liquid pressure is assumed to

$$R_0/r_i - R_0/r_j = (i - j)Bo_m \quad (2)$$

Mass (or volume) conservation gives:

$$V = V_1(\alpha_1) + \sum_{i=2}^{n-1} V_i(\alpha_i) + V_n(\alpha_n) \quad (3)$$

where $\alpha_i = \pi + \arccos(R_0/(r_i + R_1))$ is the angle measured counter-clockwise from the positive x-axis to the line connecting the center of the solid grain with the center of the *meniscus* i , as depicted in Fig. 1a. The specific expression of V_i depends on the pore-throat structure. In our system, V_i is:

$$V_i(\alpha_i) = \begin{cases} \frac{9\pi - 6\alpha_i}{2\cos^2\alpha_i}(R_0 + R_1\cos\alpha_i)^2 + (4 - 3\tan\alpha_i)R_0^2 + (3\alpha_i - 4\pi)R_1^2, i = 1, n \\ \frac{3\pi - 2\alpha_i}{\cos^2\alpha_i}(R_0 + R_1\cos\alpha_i)^2 + (4 - 2\tan\alpha_i)R_0^2 + (2\alpha_i - 3\pi)R_1^2, i = 2, 3, \dots, n-1 \end{cases} \quad (4)$$

be uniform within a pore. This *uniform pressure assumption* is rationalized in detail as shown in Supporting Information S2. We assign indices to the menisci of a linear ganglion in terms of the pore body sequence, from 1 for menisci in the most upstream occupied pore to n for menisci in the most downstream occupied pore. Thus, the meniscus in the i^{th} pore is denoted as *meniscus* i , and the uniform liquid pressure in pore i is denoted as $P_{l,i}$. The ganglion pressure P_g is also assumed to be uniform inside the entire ganglion (Ng and Payatakes, 1980; Payatakes, 1982) before mobilization, so ganglion menisci shape as circular arcs. To balance the external field, the pressure differences ($P_g - P_{l,i}$) across downstream interfaces have to be larger than those across upstream ones. Accordingly, the ganglion responds to the external field: menisci upstream retract while menisci downstream stretch, as shown in Fig. 1a.

We introduce a modified Bond number Bo_m to characterize the external field. It is defined as the ratio of the external driving force along a pore unit to the characteristic capillary pressure: $Bo_m = \frac{2|\nabla\mathcal{P}|R_0}{\gamma/R_0} = \frac{2|\nabla\mathcal{P}|R_0^2}{\gamma}$, where $|\nabla\mathcal{P}|$ represents the external pressure gradient so $2|\nabla\mathcal{P}|R_0$ is the pressure drop over a pore length, γ is the surface tension so γ/R_0 is the reference capillary pressure. If the external field is a gravity field, $|\nabla\mathcal{P}| = \Delta\rho g$, where $\Delta\rho$ is the density difference between two phases and g is the acceleration of gravity, so we obtain $Bo_m = \frac{2\Delta\rho g R_0^2}{\gamma}$ which is the classical Bond number. If the external field is a flow field, $|\nabla\mathcal{P}| = \frac{\mu v}{k_e}$, where μ is the liquid viscosity, k_e is the effective permeability and v is the superficial Darcy velocity, we obtain $Bo_m = \frac{2R_0^2}{k_e} \frac{\mu v}{\gamma} = \frac{2R_0^2}{k_e} Ca$, proportional to the classical capillary number. Here we neglect the influence of the ganglion on the external field.

2.2. Determination of ganglia status

We can express the capillary pressure equations for *meniscus* i and *meniscus* j as follows:

$$P_g - P_{l,i} = P_{c,i} = \gamma/r_i \quad (1a)$$

$$P_g - P_{l,j} = P_{c,j} = \gamma/r_j \quad (1b)$$

where we have used the Young-Laplace equation $P_c = \gamma/r$. Here, r_i is the radius of curvature of *meniscus* i . Since the external pressure gradient is assumed to be constant, we have:

$$P_{l,i} - P_{l,j} = 2(j - i)R_0|\nabla\mathcal{P}| \quad (1c)$$

By rearranging and non-dimensionalize Eq. (1a)–(1c) we obtain Eq. (2) to correlate the morphologies of different menisci:

The total potential energy F_t is defined as $F_t = F_s + W_p$, where F_s is the surface free energy of the ganglion and W_p is the work done by the ambient liquid on ganglion menisci during the ganglion evolution. The thermodynamic reason for combining F_s and W_p is explained in detail in Supporting Information S3. According to this definition, $\Delta F_t = 0$ during reversible processes, while $\Delta F_t < 0$ during irreversible processes. We note that this energy analysis is only valid before startup, as once a ganglion is started up, the total potential energy may transfer to kinetic energy with the accompanying viscous dissipation.

The surface free energy F_s is calculated as:

$$F_s = \gamma A = \gamma \left(A_1(\alpha_1) + \sum_{i=2}^{n-1} A_i(\alpha_i) + A_n(\alpha_n) \right) \quad (5)$$

where A_i in our system is

$$A_i(\alpha_i) = \begin{cases} (4\alpha_i - 6\pi)/\cos\alpha_i, i = 1, n \\ (8\pi - 6\alpha_i)R_1 + (6\alpha_i - 9\pi)(R_0/\cos\alpha_i + R_1), i = 2, 3, \dots, n-1 \end{cases} \quad (6)$$

When the ganglion volume in pore i changes by dV_i (induced by changes in Bo_m), the elemental work done by the ambient liquid on all menisci is calculated as $\delta W_p = \sum_{i=1}^n P_{l,i} dV_i$. Hence, we have:

$$W_p = \sum_{i=1}^n \int P_{l,i} dV_i \quad (7)$$

We only consider stable morphology, i.e., a ganglion must spontaneously revert to its original state after an infinitesimal perturbation. We identify two conditions where a ganglion becomes unstable (Details elaborated in (Wang et al., 2021)): (1) if the most upstream *meniscus* 1 reaches a *critical* state ($\alpha_1 = 1.25\pi$), then *flinch* (FC) happens — the ganglion tail retracts inward and occupy one fewer pore; and (2) if the most downstream *meniscus* n reaches a *full* state ($\alpha_n = \pi$), then *break-through* (BT) happens — the ganglion head stretches outward and occupy one more pore. The *critical* and *full* states are shown in Fig. 1b, which correspond to the minimum and maximum possible local capillary pressures, respectively. We make the quasi-static assumption for the ganglion evolution: when no BT or FC occurs, the ganglion deformation is reversible; during BT or FC, however, the ganglion irreversibly reconfigures.

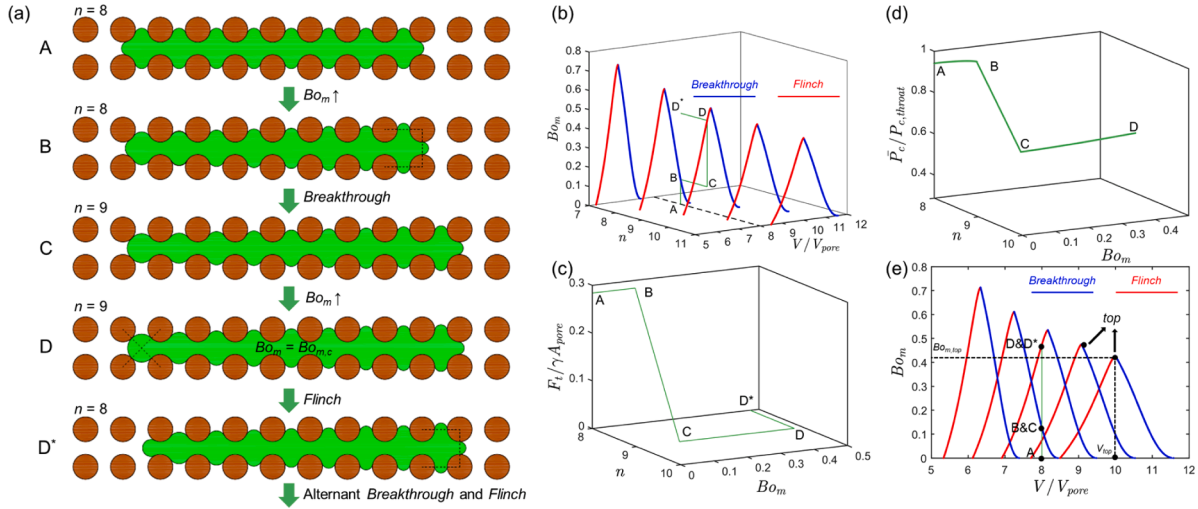


Fig. 2. (a) Morphologic evolution of ganglion at configuration A ($V = 8V_{\text{pore}}$ and $n = 8$) under increasing external field strength Bo_m . When $Bo_m < Bo_{m,c}$, the ganglion undergoes *Mode One* (reversible deformation, like A to B and C to D) and *Mode Two* (irreversible breakthrough or flinch, like B to C) alternately. When $Bo_m = Bo_{m,c}$, the ganglion starts wriggling like a worm, undergoing alternating breakthrough and flinch. Evolution of (b) the pore occupancy n , (c) the total potential energy F_t , and (d) the mean capillary pressure $\bar{P}_c$ of ganglion A under increasing external field strength Bo_m . (e) The 2-D version of Fig. 2b. In Fig. 2b & 2e, blue lines and red lines denote the breakthrough boundary and flinch boundary, $Bo_{m,\text{boundary}}(V, n)$, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3. Results and discussion

3.1. Ganglia morphology evolution before startup

3.1.1. A demonstrative case

We track the morphological changes of a ganglion under increasing Bo_m . We use the ganglion with $V = 8V_{\text{pore}}$ and $n = 8$ depicted in Fig. 2a as a demonstration. We refer to its initial configuration (at rest, $Bo_m = 0$) as configuration A. The ganglion evolution can be described as follows:

- Initially, the ganglion at configuration A undergoes reversible deformation as Bo_m increases. Specifically, the menisci upstream retracts while the menisci downstream stretch.
- When either of the unstable conditions (*critical* in the tail or *full* in the head) is met, an irreversible sudden reconfiguration occurs. For configuration A, a *full* state in the head appears first, denoted by configuration B. Further increasing Bo_m triggers a BT event, and configuration B evolves irreversibly into configuration C and becomes stably trapped again with pore occupancy increased by one.
- As Bo_m continues to increase, configuration C deforms reversibly until the next unstable condition is satisfied. Then, a *critical* state is reached (configuration D).
- Once configuration D is reached and Bo_m further increases, an FC happens, and unstable configuration D reshapes irreversibly to configuration D* with pore occupancy reduced by one. However, configuration D* is also unstable, resulting in subsequent BT that reshapes the ganglion into configuration D again. As a result, this ganglion undergoes alternating BT and FC. Consequently, the ganglion keeps moving forward, wriggling like a worm, even without any further increase in Bo_m . The ganglion starts up. The minimum external field strength required to reach configuration D, $Bo_{m,c}$, can thus be identified as the startup criterion. The corresponding state of the ganglion is named the “kick-off state”. The pore occupancy of the ganglion at the kick-off state is denoted as n_V .

To briefly summarize, before a linear ganglion starts up ($Bo_m < Bo_{m,c}$), it undergoes two modes alternately as Bo_m increases: *Mode One* involves reversible deformation, while the pore occupancy keeps constant, such as transitions from A to B and C to D; *Mode Two* involves irreversible BT or FC into another stable status that the pore occupancy

changes by one, such as transitions from B to C. These two modes alternate periodically until $Bo_{m,c}$ is reached.

Once $Bo_m = Bo_{m,c}$, the ganglion can never achieve an equilibrium configuration. It starts wriggling like a worm, undergoing alternating BT and FC. This “wriggling worm” motion is referred to as “Ameiboid motion” in visualized experiments (Ng et al., 1978). If $Bo_m \gg Bo_{m,c}$, the ganglion may undergo dynamic displacement (Payatakes, 1982; Dias and Payatakes, 2006) and flows at steady state. Identifying the specific boundary between worm-like wriggling and constant velocity steady-state flow is not in the scope of this work. A recent theoretical study, drawing an analogy between the motion of a viscous slug and a pendulum’s motion inside a fluid-filled rotating drum, may aid in understanding the transition from worm-like wriggling to achieving a constant-velocity steady-state flow (Primkulov, 2023).

Besides tracking the morphological changes of the ganglion, we also track changes in pore occupancy (in Fig. 2b), total potential energy (in Fig. 2c), and capillary pressure (in Fig. 2d). Fig. 2e is a 2-D version of Fig. 2b. It’s important to note that the capillary pressure varies along a ganglion under non-zero Bo_m (i.e. capillary pressure is constant across each free interface, but different for different free interfaces). Therefore, we calculate the arithmetic mean capillary pressure in both the head (meniscus 1) and the tail (meniscus n) to provide a visual representation, denoted as $\bar{P}_c$.

3.1.2. Boundary lines and general picture for ganglion evolution before startup

Physical picture of boundary lines. For a given ganglion, as Bo_m gradually increases, it either reaches a *full* state at the head first, or reaches a *critical* state at the tail first. Further increasing Bo_m triggers a BT event or FC event, respectively, which leads to the change of n by one. We denote the external field intensity required for the ganglion with given V and given n to reach the unstable condition (*full* state in the head or the *critical* state in the tail) as $Bo_{m,\text{boundary}}(V, n)$. Please note that the value of n here may not necessarily be the kick-off state’s pore occupancy n_V .

By enumerating all possible V and n , we obtain $Bo_{m,\text{boundary}}(V, n)$. The computed results for $Bo_{m,\text{boundary}}$ are depicted by blue and red boundary lines in Fig. 2b (and a 2-D version in Fig. 2e). The envelope in (V, n) space confined by $Bo_{m,\text{boundary}}(V, n)$ at a given n covers all possible stable states for linear ganglia in an external field.

Calculation of boundary lines. To calculate $Bo_{m,boundary}$, we first need to determine whether the ganglion experiences *BT* or *FC*. To address this question, we define a special state - the “*top*” state. The ganglion at the *top* state possesses a head in the *full* state and a tail in the *critical* state, resulting in simultaneous occurrences of both *BT* and *FC* (the intersection point of the blue and red *boundary* lines of the same n as shown in Fig. 2e). The volume of the ganglion (with the *kick-off* state’s pore occupancy n) at the *top* state is denoted as $V_{top}(n)$, and the corresponding external field strength is denoted as $Bo_{m,top}(n)$. Hence, for the ganglion with given V and given n , if $V > V_{top}(n)$, *BT* arrives earlier thus $r_n = R_0 - R_1$, while if $V < V_{top}(n)$, *FC* arrives earlier thus $r_1 = \sqrt{2}R_0 - R_1$. Utilizing Eq. (2), we have:

$$\begin{cases} Bo_{m,boundary}(V, n) = \frac{1}{n-1} \left(\frac{R_0}{r_n} - \frac{R_0}{r_1} \right) \\ r_1 = \sqrt{2}R_0 - R_1, & \text{if } V < V_{top}(n) \\ r_n = R_0 - R_1, & \text{if } V > V_{top}(n) \end{cases} \quad (8)$$

where $V_{top}(n)$ is solved by combining $Bo_{m,top}(n) = \frac{1}{n-1} \left(\frac{R_0}{R_0-R_1} - \frac{R_0}{\sqrt{2}R_0-R_1} \right)$, Eq. (3) and (4), and $\alpha_i = \pi + \arccos(R_0/(r_i + R_1))$.

Finally, combining Eq. (8) with the correlations between radii $r_1, r_2, \dots, r_n$ as given in Eq. (2), and utilizing the expressions for V as provided in Eq. (3) and (4), we can ultimately determine $Bo_{m,boundary}(V, n)$.

General picture for ganglia evolution before startup. When Bo_m is below $Bo_{m,boundary}(V, n)$, the ganglion of volume V and pore occupancy n evolves reversibly. When Bo_m increases to $Bo_{m,boundary}$, an irreversible reconfiguration occurs: if the *full* state appears first, *BT* occurs, resulting in n increasing by one; while if the *critical* state in the tail appears first, *FC* occurs, resulting in n decreasing by one.

After an irreversible reconfiguration, the state of the ganglion jumps from the *boundary* line of (V, n) to a neighboring new state (V, n^*) , where $n^* = n + 1$ or $n - 1$. If $Bo_{m,boundary}(V, n) < Bo_{m,boundary}(V, n^*)$, the new state is stable, and the ganglion will keep reversibly evolving with increasing Bo_m ; if $Bo_{m,boundary}(V, n) \geq Bo_{m,boundary}(V, n^*)$, however, there will no longer be a stable static configuration for the ganglia with volume V , so it is started up. The last stable state is the *kick-off* state, and the corresponding $Bo_{m,boundary}$ is $Bo_{m,c}$ for the given V .

3.2. Ganglion startup and thermodynamic normalization

We compare behaviors of ganglia with identical V but different initial n in an external field. Specifically, we examine ganglia A_1, A_2 , and A_3 with identical $V = 8V_{pore}$, but with initial pore occupancies of $n = 8, n = 10$, and $n = 9$, respectively, as shown in Fig. 3a.

Ganglion A_1 has been analyzed earlier and is found to undergo repeated *Mode One* and *Mode Two* until *configuration D* is reached, after which it wriggles like a worm. Ganglion A_2 follows a similar startup process. With Bo_m increasing, *configuration A₂* deforms reversibly into B_2 , then, irreversibly into C_2 through *FC*, and finally into D via reversible transformation. Once *configuration D* is reached, $Bo_m = Bo_{m,c}$, the ganglion starts up and wriggles like a worm. Ganglion A_3 , however, deforms reversibly to D , without any *BT* or *FC*. Despite their distinct startup processes, all ganglia (A_1, A_2 , and A_3) ultimately evolve to the identical *configuration D*, the *kick-off* state. For ganglia at the *kick-off* state, removing Bo_m causes them to reversibly deform back to *configuration A₃*.

During the startup process, the normalization of many properties occurs after this process:

Normalization of n . The evolution of n during the startup process for different ganglia (A_1, A_2 , and A_3) is shown in Fig. 3b. Regardless of the multiple possible initial pore occupancies, ganglia of the same V eventually normalize to one certain *kick-off* state of $n = n_V$. Rigorous proof of this claim and the calculation of n_V for any n is shown in [Supporting Information S4](#).

Normalization of F_t . We demonstrate that the *kick-off* state is of the minimum F_t at given V . Specifically, we track the change of F_t at $V = 8V_{pore}$ and $n = 8, 9, 10$ before startup as shown in Fig. 3c. We set the zero point of F_t as that of the same *kick-off* state. F_t remains constant $\Delta F_t = 0$ during *Mode One*, which is reversible, while decreases abruptly $\Delta F_t < 0$ at *Mode Two*, which is irreversible. For ganglia with the same V but different initial n , their F_t curves step-by-step reduce and overlap as Bo_m increases, and eventually merge into the *kick-off* state with the minimum total potential energy $F_{t,min}$.

Normalization of P_c . We also track the mean capillary pressure $\bar{P}_c$ of the ganglia before startup, as illustrated in Fig. 3d. As Bo_m increases, $\bar{P}_c$ gradually moves away from both $P_{c,full}$ and $P_{c,critical}$, and eventually converging towards the mean capillary pressure of the *kick-off* state.

So far, we have demonstrated that n, P_c , and F_t of ganglia normalize

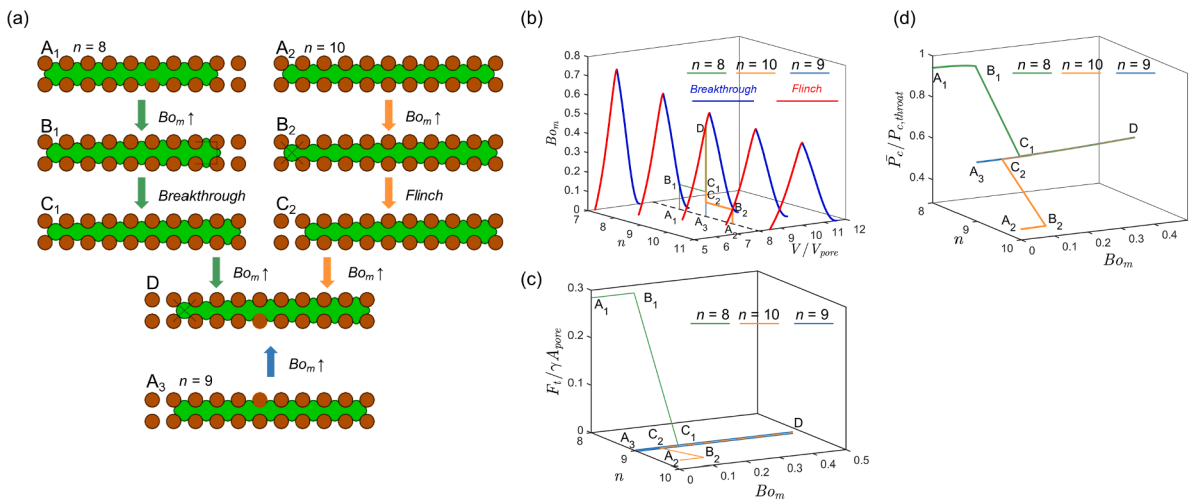


Fig. 3. (a) Evolution of ganglia of the same size $V = 8V_{pore}$ but different initial pore occupancies (A_1, A_2 , and A_3) under increasing Bo_m . Although these ganglia have different initial shapes, once starting up, they gradually normalize to the *kick-off* state ganglion D . The green arrows in Fig. 3a and the green lines in Fig. 3b, 3c, and 3d represent the evolution of the ganglion with an initial pore occupancy of $n = 8$, progressing from A_1 to B_1 , then to C_1 , and finally to D . The orange arrows in Fig. 3a and the orange lines in Fig. 3b, 3c, and 3d represent the evolution of the ganglion with an initial pore occupancy of $n = 10$, progressing from A_2 to B_2 , then to C_2 , and finally to D . The blue arrows in Fig. 3a and the blue lines in Fig. 3b, 3c, and 3d represent the evolution of the ganglion with an initial pore occupancy of $n = 9$, progressing from A_3 to D . Evolution of (b) the pore occupancy n , (c) the total potential energy F_t and (d) the mean capillary pressure $\bar{P}_c$ under increasing Bo_m . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

during startup. Therefore, compared to trapped static ganglia in porous media, fewer parameters are required to adequately characterize a ganglion. In other words, startup significantly reduces ganglia's hysteresis effects. We will discuss this in more detail in the next subsection.

3.3. Reduction of capillary hysteresis

Capillary hysteresis is a critical phenomenon that challenges the modeling of multiphase fluid behaviors in porous media (Haines, 2009; Morrow and Harris, 1965; Hilfer, 2006). The P_c - S_w curves during fluid loading and unloading at the scale above REV are closely related to the saturation history, indicating the existence of multistability (Everett and Whitton, 1952; Poulovassilis, 1962; Cueto-Felgueroso and Juanes, 2016; Helland et al., 2021). Many studies handle capillary hysteresis (Everett and Whitton, 1952; Poulovassilis, 1962; Brooks and Corey, 1966; Cihan, 2014) by introducing parameters to fit the empirical relationship between P_c and S_w . However, if we briefly introduce parameters to fit the empirical model, we leave the question of missing state variables unaddressed (Schlüter, 2016). Other studies focus on identifying the missing state variables to resolve hysteresis (McClure, 2016; Schlüter, 2016; Hassanizadeh and Gray, 1993; Miller, 2019). Following the proposal of the *HG hypothesis* (Hassanizadeh and Gray, 1993), several studies employ different approaches to demonstrate that the inclusion of specific interfacial area (a_{wn}) and Euler characteristic (χ) in the P_c - S_w relationship can erase capillary hysteresis (McClure, 2016; Wang et al., 2021; Miller, 2019; Armstrong, 2016; McClure, 2018). In a d -dimensional system, $d + 1$ Minkowski functionals are required to fully characterize the status of two-phase immiscible fluids (Hadwiger, 1957; McClure, 2018; Armstrong, 2018). In our 2-D system, P_c can be expressed as a function of three functionals: V , A (or n , equivalent), and χ . Unfortunately, these parameters are hard to identify in practical applications.

Nevertheless, the origin of capillary hysteresis – multistability, is removed when a ganglion has experienced startup, as discussed in the earlier subsection. Regardless of initial morphology and thermodynamic features, a ganglion evolves to the *kick-off state* once started up. Linear ganglia of one given volume corresponds to one certain *kick-off state*. Even when the external field is removed and the ganglion is trapped again, this critical state is “remembered” that n does not change. Therefore, it is natural to suspect that the startup can reduce, or even “erase”, the capillary hysteresis effect of ganglia, by eliminating multistability. In [supporting information S5](#), we provide a visual description of the hysteresis of ganglia at the pore scale by loading and uploading

external field strength Bo_m .

Here, we demonstrate that, once a ganglion has experienced startup (i.e., the ganglion first started up and subsequently immobilized by withdrawing of the external field $Bo_m = 0$), multistability is removed thus capillary hysteresis is erased, as capillary pressure (P_c), surface free energy (F_s) and Euler characteristic (χ) become univariant functions of ganglia volume V .

Without experiencing startup, the capillary pressure of a trapped ganglion in the absence of an external field may fall into a large range between $[\gamma/(\sqrt{2}R_0 - R_1), \gamma/(R_0 - R_1)]$ (Wang et al., 2021), corresponding to the gray lines in Fig. 4a. In our specific case the pore-throat ratio is 6, this range is as large as 71 % of the maximum P_c . Nevertheless, once startup, the multistability is reduced to monostability and the P_c variation range of ganglia is largely reduced, as shown by the black lines in Fig. 4a. In Fig. 4a, the black lines correspond to the ganglion that has once experienced startup. In our specific case, the P_c variation range of black lines can be reduced to only 5 % when $n = 15$. This range becomes even smaller with increasing V , which ultimately converges to a constant $P_{c, equal}$. $P_{c, equal}$ is independent of the initial ganglion volume and morphology and is solely determined by the porous geometry. When the pore-throat ratio is 6, $P_{c, equal}$ equals $0.63\gamma/(R_0 - R_1)$. This result is an observation based on our numerical calculations; we defer the theoretical rationalization of this observation to future work.

We also observe the hysteresis reduction of surface free energy (F_s). As depicted by the gray lines in Fig. 4b, the relationship between F_s and V for a linear ganglion is complicated: for a given V , there isn't a single, unique F_s value. However, if the ganglion has once experienced startup, there is only one certain surface free energy at the *kick-off state* for a known V , represented by the black lines in Fig. 4b. Through the observation of F_i in Fig. 3c and F_s in Fig. 4b, it is evident that all relatively high-energy metastable states become unstable before startup, so the historical effects and multi-stability are eradicated. Ganglion startup thus helps to identify the state of the lowest energy for a ganglion.

For ganglia with complex shapes with branches ($\chi < 1$), startup also reduces the hysteresis. If the ganglion does not split up, it evolves to the same *kick-off state* as a linear ganglion of $\chi = 1$; if the ganglion splits up, the final state of each daughter ganglion would be of $\chi = 1$ and linear at the corresponding *kick-off state*. Details of the fission mechanism of the ganglion are shown in [Supporting Information S1](#), which well echoes previous experiments and simulations (Ng and Payatakes, 1980; Payatakes, 1982).

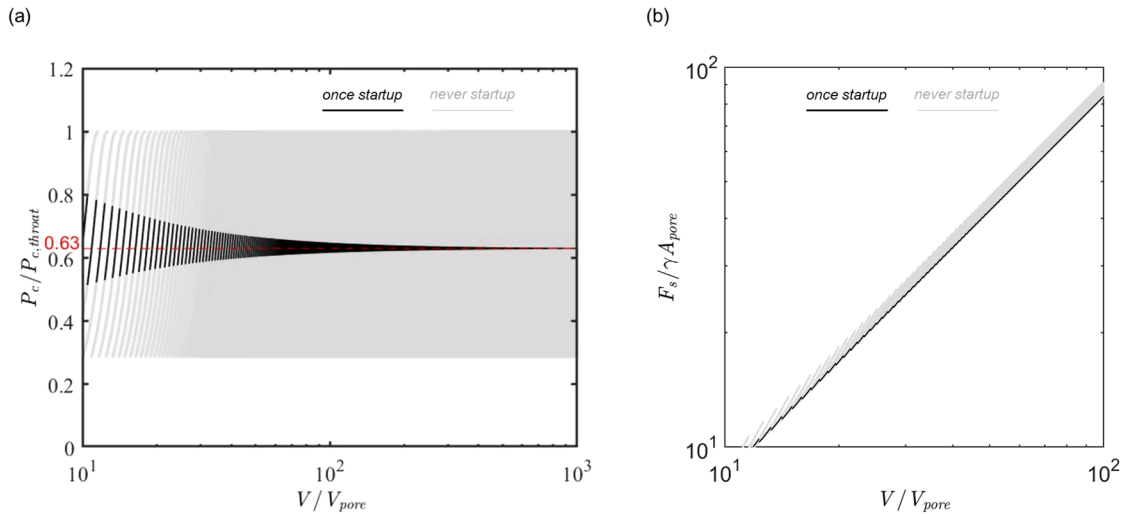


Fig. 4. Relationship between (a) capillary pressure P_c and volume V , (b) surface free energy F_s and volume of ganglia V . Both gray lines and black lines correspond to static trapped ganglia under $Bo_m = 0$. However, black lines represent ganglia which have once experienced startup, while gray lines represent ganglia which have never experienced startup.

In a previous study, we have shown one (approximately) constant functional “the specific surface area A/V ” (Wang et al., 2021). Now, we offer the other two, namely P_c and χ , for large ganglia if they experienced startup in porous media. Therefore, we can simply use one variable, such as V , to describe the thermodynamic properties of large ganglia after startup in the 2-D porous medium. Ganglion startup thus effectively reduces local capillary hysteresis in porous media, which may highly simplify the modeling of residual oil mobilization & trapping in petroleum engineering, bubble rising during soil remediation, and natural gas primary migration in the subsurface environment.

3.4. Criteria of ganglia startup

In earlier literature, the criterion for ganglia startup is mostly characterized by ganglia length as it is assumed proportional to external driving force (Larson et al., 1977; Datta et al., 2014; Larson et al., 1981; Chatzis and Morrow, 1984). However, as the ganglion length can vary before being fully mobilized, what these literature refer to as “ganglion length” is actually not the initial ganglion length before mobilization, but “the length of the ganglion after mobilization”, which was not fully resolved before. Therefore, ganglion length may not be a suitable variable in this context of ganglia startup.

Fortunately, the reduction of hysteresis by ganglia startup discussed above allows us to determine the criteria of ganglia startup, $Bo_{m,c}$, as a univariate function of volume V , as different ganglia with the same V normalize into identical morphology at the moment of startup regardless of their initial morphologies. Utilizing the *boundary lines* in section 3.1.2, we set the ganglion at the *kick-off state* $n = n_V$ and obtain:

$$Bo_{m,c} = Bo_{m,boundary}(V, n = n_V), \quad (9)$$

as shown by the black lines in Fig. 5. In fact, the black lines in Fig. 5 are precisely the “maximum value envelope curve” of the blue and red

boundary lines in Fig. 2e (with x-y axes exchanged). The zoom-in of the serrated structure in Fig. 5 is presented in supporting information S4.

In Fig. 5, the ganglion behaviors are divided by the black lines into two regimes:

- *Regime 1*: The ganglion is trapped at equilibrium. In this regime, hysteresis effects are significant in that one specific volume corresponds to multiple metastable morphologies. Increasing the external field strength in this regime may trigger irreversible *BT* and *FC* that alternates the pore occupancy, without mobilizing the ganglion.
- *Regime 2*: the ganglion starts. When the ganglion status is close to the criterion line, it behaves wriggling like a worm with alternating *BT* and *FC*, that an actual “steady state” cannot be achieved; when the ganglion is far from the criterion line, the fluctuation may be compressed so that the ganglion flows with constant velocity, which is extensively discussed in earlier publications.

4. Limitations

Our conceptual model is admittedly limited by several assumptions. First, the roles of heterogeneity and polydispersity in pore sizes are not investigated, which likely have a significant impact on ganglion growth, shrinkage, and hysteresis (Cueto-Felgueroso and Juanes, 2016; Joekar-Niasar, 2013). Second, the model is limited to 2-D for simplicity, following our previous work. Third, ganglia are assumed to be completely non-wetting, and the external pressure gradient is assumed to be constant for simplicity. Fourth, we do not consider ganglion behavior after startup, where unstable interfaces emerge, requiring more complicated hydrodynamic analysis. Despite these limitations, we believe our simple analysis successfully captures the essence of how ganglia evolve in an external field and how the startup can reduce or even erase local hysteresis effects. Future work should attempt to relax the above assumptions.

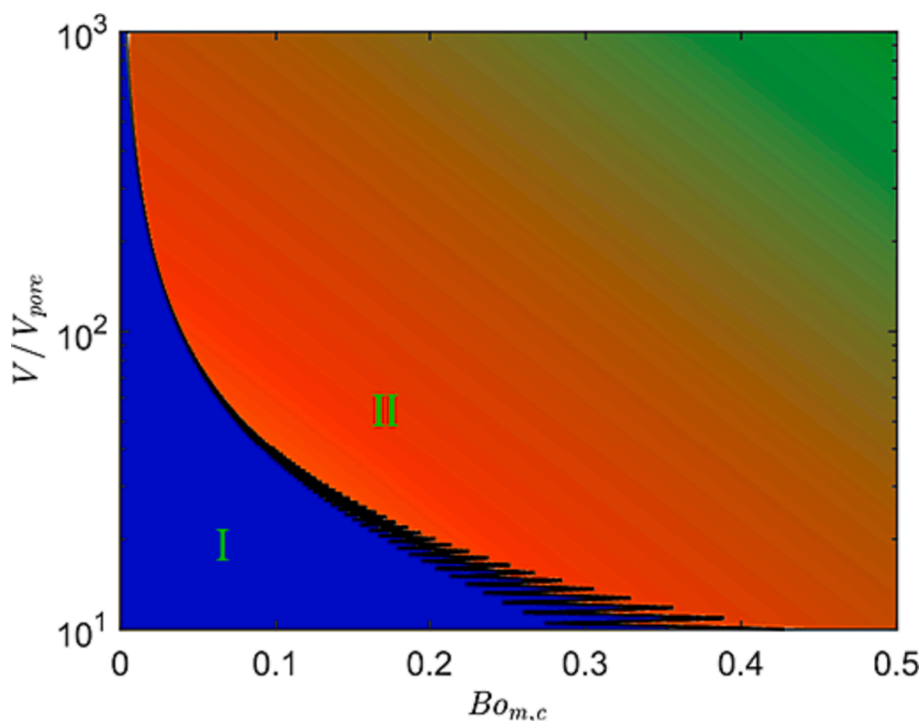


Fig. 5. Three regimes for the ganglion evolution under an external pressure gradient. Black lines denote the minimum external field strength, $Bo_{m,c}$, to mobilize a ganglion, as a function of the ganglion volume V . *Regime 1* corresponds to a static ganglion, depicted in blue. In this regime, the ganglion remains stationary and does not exhibit significant movement. *Regime 2* represents a wriggling ganglion, depicted in red. Ganglia in this regime exhibit wriggling behavior, characterized by periodic movements and shape changes. *Regime 3* depicts a dynamic ganglion, which also exhibits wriggling behavior, but with higher intensity compared to *Regime 2*. The transition between *Regime 2* and *Regime 3* is illustrated with a color gradient ranging from red to green. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5. Conclusion

We track the ganglion startup process under an external field in a 2-D uniform porous medium and investigate the evolution of ganglion morphology and thermodynamic features. We show that the ganglion startup process in porous media is more complicated than presumed before.

A trapped ganglion of a certain volume has multiple possible configurations. The external force may trigger irreversible reconfiguration of a trapped ganglion, although it may not be strong enough to mobilize the ganglion. With increasing Bo_m , the ganglion undergoes reversible deformation (pore occupancy unchanged) and irreversible *BT* or *FC* (pore occupancy changed by one) events alternately, until the external field strength reaches a critical value, $Bo_{m,c}$, that fully mobilizes the ganglion. When a ganglion is just mobilized under $Bo_{m,c}$, it does not move at a steady state. Instead, it starts wriggling like a worm, alternating between two non-equilibrium states. This wriggling moving mode is rigorously rationalized and agrees with experimental observation (Ng et al., 1978).

We show that ganglia mobilization largely reduces capillary hysteresis. Normalization of n , F_t and P_c emerges during ganglion startup: although linear ganglia with the same V may have various morphology at rest, all possible initial states normalize to a *kick-off state* with the same pore occupancy n_V and the minimum total potential energy F_t once they start up. For large linear ganglia, the *kick-off state* shares almost the same P_c regardless of V , which depends only on the porous geometry. Due to this reduction of hysteresis, we can analytically resolve the startup criterion, $Bo_{m,c}$, as a univariate function of V , regardless of many different initial configurations at the same V .

The capillary hysteresis reduction and the determination of the startup criterion may largely simplify the modeling of dispersed fluid behaviors in porous media, which helps future studies in CO_2 sequestration, DNAPL remediation, hydrogen storage, and enhanced oil recovery.

CRedit authorship contribution statement

Chuanxi Wang: Writing – original draft, Investigation, Formal analysis, Methodology. **Ke Xu:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Investigation, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.ces.2024.119982>.

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